Subject: Mathematics **Class: 12th**
Max. Marks: 100 **Time: 3 Hrs**

Long Answer Questions

Q1. Define symmetric and skew symmetric matrices. Prove that every square can be expressed as a sum of symmetric and skew symmetric matrices.

Or

Define determinant of a matrix and using the properties of determinants
Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$$ **06 marks**

Q2. Define differentiability and continuity of a function. What is the relationship between them, justify your answer.

Or

If $(x - a)^2 + (y - b)^2 = c^2$ for some c > 0, prove that

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$, is a constant independent of *a* and b! **6 marks**

Q3. Evaluate $\int e^{ax} \sin bx \; dx$

Hence, deduce $I = \frac{e^{ax}}{\sqrt{a^2+b^2}} \sin\left(bx + \tan^{-1} \frac{b}{a}\right) + c$

Or

Define definite integral of a function and find the area under y = $(x + c^{2x})$ between the limits 0 and 4: **6 marks**

Q4. Determine graphically the minimum value of the objective function Z=-50x+20y subject to the constraints.
2x-y≥-5, 3x+y≥3, 2x-3y≤12, x≥0, y≥0

Or

One kind of cake requires 200 kg of flour and 25g of fat and another kind of requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5Kg of flour and 1Kg of ft assuming that there is no shortage of the other ingredients used in making the cakes. **6 marks**

Q5. Define skew lines. Find the shortest distance between two skew lines with equations

$r = \vec{a}_1 + \pi \vec{b}_1$ and

$r = \vec{a}_2 + \mu \vec{b}_2$ in vector form

Or

For any two vectors $\vec{a}$ *and* $\vec{b}$

$\left|\vec{a}+\vec{b}\right| \le \left|\vec{a}\right| + \left|\vec{b}\right|$ prove it and if $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}\right| + \left|\vec{b}\right|$, than what happens?

6 marks

Short Answer Type Questions

Q6 Define bijective function and show that the function
F : R → R defined by $f(x) = x^3 + 1$ is a bijection. **4 marks**

Q7. Define reflexive, symmetric and transitive relation with an example to each.
4 marks

Q8. Using elementary transformation. Find the inverse of $\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix}$. **4 marks**

Q9. State Lagrange's. Mean value theorem and interpret geometrically. **4 marks**

Q10 The side of a square sheet of metal is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long? **4 marks**

Q11. Evaluate $\int \frac{1}{1+\tan x}\,dx$ **4 marks**

Q12 Find the equation of tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_0, y_0).

4 marks

Q13. Find the equation of the plane passing through the points (1, 2, 3) and perpendicular to the plane

$2x + 3y + 4z - 5 = 0$

$4x + 6y + 8z - 15 = 0$ **4 marks**

Q14. Prove by vector method an angle in a semi circle is a right angle. **4 marks**

Q15. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black. 4 marks

Very Short Answer Type

Q16. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x. **2 marks**

Q17. Find x and y if,

$$2\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} y & 0 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix}$$

2 marks

Q18. Is the function defined by

$f(x) = \begin{cases} x+5, & if \;\; x \le 1 \\ x-5 & if \;\; x > 1 \end{cases}$, a continuous function **2 marks**

Q19. Find $\frac{dy}{dx}$ if $2x + 3y = \sin y$ **2 marks**

Q20. Evaluate $\int \sec x \; dx$ **2 marks**

Q21. State 2^{nd} fundamental theorem of integral calculus. **2 marks**

Q22. Solve $xy \frac{dy}{dx} = e^x$ **2 marks**

Q23. Prove that the scalar product between the given vector is commutative

$\vec{a} = a_1 \hat{i} + a_2 \hat{j}$ $\forall$ $a_1 b_1 \in R$

$\vec{b} = b_1 \hat{i} + b_2 \hat{j}$ $a_2 b_2 \in R$ **2 marks**

Q24. A die is rolled, if the outcome is an even number. What is the probability that it is prime number? **2 marks**

Q25. Evaluate P (A $\cup$ B)

If $2P(A) = P(B) = \frac{5}{13}$

and $P\left(\frac{A}{B}\right) = \frac{2}{5}$ **2 marks**

Objective Type questions

Q26. When two coins are tossed, what is the probability of at most two heads?

a) 1

b) -1

c) 0

d) None of this **1 mark**

Q27. If A and B are two n × n non-singular matrices, then

a) AB is non-singular b) AB is singular

c) $(AB)^{-1} = A^{-1} B^{-1}$ d) $(AB)^{-1}$ does not exist **1 mark**

Q28. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \infty$ then $\frac{dy}{dx} =$

a) 0 b) 1 c) x d) y 1 mark

Q29. $\frac{d}{dx}(\tan^{-1} x + \cos^{-1} x)$ is equal to

a) $\frac{1}{1+x^2}$ b) 0 c) $\frac{-1}{1+x^2}$ d) None of these 1 mark

Q30. $\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

a) tan$x$ + cot x + c b) tan$x$ – cot$x$ + c
c) tan x + cot 2x + c d) tan x – cot 2x + c 1 mark

Q31. $\int \frac{1}{e^x + 1}\, dx$ is equal to

a) log $(e^x + 1)$ + c b) log $(1 + e^x)$ + c
c) – log $(1 + e^{-x})$ + c d) None of these 1 mark

Q32. The differential equation of all circles of radius a is of order
a) 2 b) 3 c) 4 d) none of these 1 mark

Q33. | a × b | is equal to
a) ab sin θ b) ab cosθ
c) ab sinθ $\hat{n}$ d) none of these 1 mark

Q34. Solution of the differential equation Xdy-Ydx =0 represents
a. A rectangular hyperbola
b. A straight line passing through origin
c. Parabola whose vertex is at the origin
d. Circle whose centre is at the origin

1 mark

Q35. If A + B are independent events P(A ∪ B) is equal to
a) P(A) + P(B) b) P(A) + P(B) – P(A∩B)
c) P(A) . P(B) d) None of these 1 mark